ECON 714A: Macroeconomic Theory - Handout 7

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1 A Model of Firm Dynamics

We go through the model of firm dynamics in Hopenhayn and Rogerson (1993). The economy consists of incumbent and entrant firms where incumbent firms decide to stay or exit, and entrant firms decide to enter or not. Cost of staying in the market is k and the entry cost is k_e. Each entrant firm draws an idiosyncratic productivity from a from the distribution G. Let $z = \{p_t, w_t\}_{t=0}^{\infty}$ be the sequence of prices.

1.1 Incumbent's Problem

An incumbent firm maximizes the present discounted value of its profits.

$$v_t(a, z) = \pi(a, p_t, w_t) + \frac{1}{R} \max \left\{ 0, \int v_{t+1}(a', z) dF(a'|a) \right\}$$

From the second term, we have that an exit threshold $a_t^*(z)$ exists such that the firm exits if $a_t < a_t^*(z)$. Also, we have that

$$\int v_{t+1}(a', z) dF(a'|a^*) = 0$$

1.2 Entrant's Problem

An entrant firm pays k_e to enter the market, and it begins production the next period. It draws productivity from distribution G, so it does not enter the market if the expected present value of profits is lower than the entry cost. Hence, the entry condition is written as follows.

$$\frac{1}{R} \int v_{t+1}(a', z) dG(a') \leq k_e$$

Denote m_t as the mass of entrants at time t. $m_t \geq 0$ when the entry condition above holds, with $m_t > 0$ whenever the equality holds. In equilibrium, if $\frac{1}{R} \int v_{t+1}(a', z) dG(a') > k_e$, all firms will continue entering the market until z is adjusted to equate the cost and benefit of entering the market.

Hence, the entrant's problem is written as follows. It enters the market if $v_t^e(z) \geq 0$.

$$v_t^e(z) = -k_e + \frac{1}{R} \max \left\{ 0, \int v_{t+1}(a', z) dG(a') \right\}$$

1.3 Distribution of Firms

Let $\mu_t(\mathcal{A})$ be the measure of incumbents with productivity $a \in \mathcal{A}$. μ_t is the state of the economy, and it is an endogenous object that evolves over time. Law of motion for μ_t could be written as follows.

$$\mu_{t+1}([0, a']) = \int \mathbb{I}(a \geq a_t^*) F(a'|a) d\mu_t(a) + m_{t+1} G(a')$$

1.4 Market Clearing

Assume that the industry demand $D(p_t)$ is exogenously given and the industry supply is endogenous: $Y_t = \int y(a, p_t, w_t) d\mu_t(a)$. Goods market is cleared with $Y_t = D(p_t)$. In this setting, we normalize $w_t = 1$.

1.5 Equilibrium

Given an initial distribution μ_0, a perfect foresight equilibrium consists of sequences $\{p_t, m_t, a_t^*, \mu_t\}_{t=0}^{\infty}$ such that

1. Goods market clears.
2. Incumbent firms make optimal exit decisions.
3. Entrant firms make optimal entry decisions.
4. Distribution μ_{t+1} follows the law of motion.

We focus on the stationary equilibrium of this perfect foresight equilibrium that consists of (p, m, a^*, μ) with $\mu_{t+1} = \mu_t, \forall t$.

1.6 Comparative Statics

Consider an increase in entry cost k_e. This increases the value $v(a, p)$ of incumbent for all a by free-entry condition. Also, this implies that the price p goes up in the incumbent's problem. With higher $v(a, p)$ and p, the exit threshold a^* for firm productivity goes down and average age of firms increase. Firms' ages going up implies that there are less entry/exit occurring in the market, so m is lower. Lastly, the overall effect on output is ambiguous because firms are now less productive on average as a^* is lower but the employment is higher because of higher p.

2 Computation Exercise

We now study the computation algorithm to solve for the equilibrium price p and exit threshold a^* for different levels of k_e when the distribution of productivity F and G and the distribution of firms μ is given.

Algorithm 1: Stationary Partial Equilibrium of Hopenhayn and Rogerson (1993)

Set up parameters F, G, μ, and m, R with discretization
Choose a value of p
Make a initial guess of value functions $v(a, p)$
while *error* $>$ *tol* **do**
  foreach a **do**
    $v_{new}(a, p) = \pi(a, p, 1) + \max\{0,\ \sum_{a'} v_{old}(a', p) dF(a'|a)\}$
  end
  error $= \max |v_{new}(a, p) - v_{old}(a, p)|$
  $v_{old}(a, p) = v_{new}(a, p)$
end
Find a^* such that $\sum_{a'} v(a', p) dF(a'|a^*) = 0$
Return $v(a, p)$ and a^*
Find k_e such that $\frac{1}{R} \sum_{a'} v(a', p) dG(a') = k_e$

References

Hopenhayn, H., & Rogerson, R. (1993). Job turnover and policy evaluation: A general equilibrium analysis. *Journal of Political Economy*, *101*(5), 915–938.