Note 1: Numerical Methods for Macroeconomists Ch.4[*]

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1 Environment

There are households, firms, and a government in each country. Households maximize utility by choosing consumption c and working hours h. Output technology o follows a Cobb-Douglas production function $o = zk^{\theta}h^{1-\theta}$ and the capital stock k is assumed to be fixed. (This is a static model.) Firms maximize profit based on this technology, and investment i is assumed match the amount of capital depreciation δk.

The government taxes consumption and labor income to buy government produced goods g and make lump-sum transfer λ to the households. Hence, the government budget constraint is given as follows.

$$\lambda + g = \tau_c c + \tau_h wh$$

Lastly, resource constraint in each country is given as follows.

$$c + g + \delta k = o$$

1.1 Households

A household in each country maximizes utility by consuming c and working h. An assumption is that the working hours cannot exceed 100 hours per week. The utility maximization problem could be written as:

$$\begin{aligned} &\max_{c,h} \log c + \alpha \log (100 - h) \\ &\text{s.t. } (1+\tau_c)c = (1-\tau_h)wh + rk + \lambda \end{aligned}$$

Solving for the first-order conditions, we get the following. For ease of notation, we define the effective tax on labor τ. This is because both consumption tax and labor income tax distorts the households' labor supply choice.

$$\begin{aligned} &\max_{h} \log \frac{(1-\tau_h)wh + rk + \lambda}{1+\tau_c} + \alpha \log (100 - h) \\ &\Rightarrow \frac{1-\tau_h}{1+\tau_c}\frac{w}{c} = \frac{\alpha}{100-h} \\ &\iff (1-\tau)\frac{w}{c} = \frac{\alpha}{100-h} \qquad \text{where } \tau \equiv \frac{\tau_c + \tau_h}{1+\tau_c} \end{aligned} \tag{1}$$

[*]This note closely follows Greenwood and Marto (2022).

1.2 Firms

Firms' profit maximization problem is given as follows. Since the supply of capital is fixed, we just focus on the firms' labor demand. We obtain that the labor share of income is $1-\theta$.

$$\begin{aligned} &\max_{h,k} zk^\theta h^{1-\theta} - wh - rk \\ &\Rightarrow (1-\theta)zk^\theta h^{-\theta} = w \\ &\iff wh = (1-\theta)o \end{aligned} \tag{2}$$

1.3 Equilibrium

We focus on the labor market equilibrium. Plug in (2) to (1) and we get the following.

$$\begin{aligned} \frac{(1-\tau)(1-\theta)o/h}{c} &= \frac{\alpha}{100-h} \\ h &= \frac{100(1-\theta)}{\alpha(c/o)/(1-\tau)+(1-\theta)} \\ \log h &= \log 100 + \log(1-\theta) - \log\left(1-\theta+\frac{\alpha}{1-\tau}\frac{c}{o}\right) \end{aligned}$$

We obtain the elasticity of labor with respect to a tax change from the last equation.

$$(1-\tau)\frac{\partial \log h}{\partial \tau} = -\frac{1}{\alpha(c/o)/(1-\tau)+1-\theta}\frac{\alpha}{1-\tau}\frac{c}{o}$$

With a given level of α and θ, h can be predicted by c/o and τ. We focus on special cases in which $\theta=0$, assuming a linear production function with no capital.

1.3.1 Case of $g=0$

When the government transfers all of its tax revenue back to the household, then $c = wh = o$ and the working hours h is $\frac{100}{\alpha/(1-\tau)+1}$. Choice of h is irrelevant to the household income in this case, so the tax distorts working hours by inducing a substitution effect.

1.3.2 Case of $\lambda=0$

When the government spends all of its tax revenue as government spending g and transfers nothing to the household, combining the household budget constraint and government budget constraint, $g=\tau wh$ and $c=(1-\tau)wh$.

$$\begin{aligned} &(1+\tau_c)c = (1-\tau_h)wh && \text{HH budget constraint} \\ \Rightarrow\ &g = \frac{\tau_c-\tau_c\tau_h}{1+\tau_c}wh + \tau_h wh = \tau wh && \text{Government budget constraint} \end{aligned}$$

Again, $c=(1-\tau)wh=(1-\tau)o$, so $h=\frac{100}{\alpha+1}$. In this case, working hours is irrelevant of tax because the income and substitution effect cancel out.

1.3.3 Case of valued government spending and no transfers

Consider an alternative utility function $\log(c + \xi g) + \alpha \log(100 - h)$ with $0 \le \xi \le 1$ and $\lambda = 0$. First order condition of the household's problem yields

$$\begin{aligned}
100(1-\tau)w - (1-\tau)wh &= \alpha(1-\tau)wh + \alpha\xi g \\
100(1-\tau)o - (1-\tau)oh &= \alpha(c+\xi g)h \\
(100-h)(1-\tau)o &= \alpha(1-(1-\xi)\tau)oh \\
h &= \frac{100}{1 + \frac{\alpha(1-(1-\xi)\tau)}{1-\tau}}
\end{aligned}$$

Cases 1.3.1 and 1.3.2 correspond to $\xi = 1$ and $\xi = 0$, respectively. Intuitively, this is because when $\xi = 1$, government spending is a perfect substitute for c so it is equivalent to lump-sum transfers. If $\xi = 0$, that means government spending is valueless to consumers so it is completely deadweight loss.

2 Mapping the Model into the Data

The model is mapped into the National Income and Product Accounts (NIPA) data. Consumption and investment spending in the data include indirect taxes IT_c and IT_i, respectively, so that should be adjusted when matching the variables in the model and the data. Capital letters below denote variables from the data.

- **Consumption Tax.** Consumption is approximately 2/3 of aggregate private spending, so 2/3 of indirect taxes falls on consumption. The other 1/3 of indirect taxes is attributed to business taxes that would be split between consumption and investment. Considering that, IT_c and τ_c can be calculated as below.

$$\begin{aligned}
IT_c &= \left(\frac{2}{3} + \frac{1}{3}\frac{C}{C+I}\right) IT \\
\tau_c &= \frac{IT_c}{C - IT_c}
\end{aligned}$$

- **Labor Income Tax.** τ_h in the model is calculated from labor income tax τ_{inc} and social security tax τ_{ss} in the data.

$$\begin{aligned}
\tau_{inc} &= \frac{\text{Direct Taxes}}{\text{GDP - IT - Capital Consumption Allowance}} \\
\tau_{ss} &= \frac{\text{Social Security Taxes}}{(1-\theta)(GDP - IT)} \\
\tau_h &= \tau_{ss} + 1.6\tau_{inc}
\end{aligned}$$

- **Consumption and Output.** Consumption in the model c is obtained by adjusting for indirect taxes on consumption and substitutable government spending $G - G_{military}$, which is non-military government spending. Output in the model o is obtained by adjusting for indirect taxes.

$$c = C + G - G_{military} - IT_c$$
$$o = O - IT$$

- **Parameters α and θ.** Labor disutility parameter α is picked to match average labor supply across countries, and it is 1.54. Labor share of income $1 - \theta$ is usually measured as

$$1 - \theta = \frac{\text{Compensation of Employees}}{NI - \text{Proprietor's Income}}$$

but, $\theta = 0.32$ is also commonly used.

3 Analysis

3.1 Actual and Predicted Labor Supply

Using the model, we predict the labor supply in terms of working hours across G7 countries. The model fits the data better for the period 1993-96 than the period 1970-74. It seems that an increase in τ is associated with a fall in h across countries.

Actual and Predicted labor Supply

		Labor Supply		*Differences*	*Prediction Factors*	
		Actual	Predicted		τ	c/o
1993–96	Germany	19.3	19.5	0.2	.59	.74
	France	17.5	19.5	2.0	.59	.74
	Italy	16.5	18.8	2.3	.64	.69
	Canada	22.9	21.3	-1.6	.52	.77
	United Kingdom	22.8	22.8	0	.44	.83
	Japan	27.0	29.0	2.0	.37	.68
	United States	25.9	24.6	-1.3	.40	.81
1970–74	Germany	24.6	24.6	0	.52	.66
	France	24.4	25.4	1.0	.49	.66
	Italy	19.2	28.3	9.1	.41	.66
	Canada	22.2	25.6	3.4	.44	.72
	United Kingdom	25.9	24.0	-1.9	.45	.77
	Japan	29.8	35.8	6.0	.25	.60
	United States	23.5	26.4	2.9	.40	.74

Figure 1: Model and Data Comparison

3.2 Financing Social Security with Private Accounts

Suppose that the social security is funded not by the government transfer, but by private retirement accounts that households save out of their income. In this case, tax does not distort agents' labor supply choice.

Consider a two-period economy where an agent works in the first period and retires in the second period. Assume a constant interest rate r and some social security τ_{ss} with lump-sum transfer λ that households receive when they retire. Then a representative households' intertemporal budget constraint would be

$$c_1 + \frac{c_2}{1+r} = (1-\tau_{ss})wh + \frac{\lambda}{1+r}$$

In this economy, tax still distorts households' labor supply choice because it decreases the after-tax wages. Government budget constraint is constructed as $\lambda = (1+r)\tau_{ss}wh$.

If retirement pensions are funded from private savings in period 1, households recognize that $\lambda = (1+r)\tau_{ss}wh$ and internalize this in its utility maximization. Then the household budget constraint changes to

$$c_1 + \frac{c_2}{1+r} = wh$$

and τ_{ss} does not enter the household's problem. Hence, there is no tax distortion with a system private retirement savings accounts.

References

Greenwood, J., & Marto, R. (2022). *Numerical methods for macroeconomists*. Economie d'Avant Garde.