Note 2: Burdett-Judd Model of Price Dispersion [*]

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1 Environment

We observe price dispersion for a single good in reality. To explain this, Burdett and Judd (1983) construct a model that generates price dispersion with homogeneous buyers and sellers. Each consumer buys one good through what Burdett and Judd (1983) call "noisy search." Buyers are homogeneous *ex ante*, but they receive different numbers of offers from sellers, and they choose the offer with the lowest price if that price is below their reservation price. Otherwise, they do not buy the good and wait for next period. The difference in the number of offers that each consumer gets generates heterogeneity *ex post* which induces price dispersion in the equilibrium.

1.1 Consumers

A consumer buys a good for a minimum price out of offers he gets. By paying for search cost c each period, he gets either one price offer with probability q or two different price offers with probability $1-q$, where each price offer p is drawn from a distribution G, i.i.d. G is defined for $\underline{p} \leq p \leq B$ and satisfies $G(B) = 1$ and $G(\underline{p}) = 0$. Then the probability distribution H of the lowest price that is offered to a consumer can be constructed as follows.

$$\begin{aligned} H(p) &= q\Pr(p_1 \leq p) + (1-q)\Pr(\min(p_1, p_2) \leq p) \\ &= qG(p) + (1-q)\left(1 - (1-G(p))^2\right) \end{aligned} \tag{1}$$

Then the expected price $v(p)$ that a consumer pays is defined as below. The second term inside the parenthesis is the expected price that the consumer will pay next period by not buying today and waiting for the next price offer.

$$\begin{aligned} v(p) &= \min\left(p, c + \int_{\underline{p}}^{B} v(p')dH(p')\right) \\ &= \begin{cases} p & \text{if } p \leq \tilde{p} \\ c + \int_{\underline{p}}^{B} v(p')dH(p') = \tilde{p} & \text{if } p > \tilde{p} \end{cases} \end{aligned}$$

[*] This note follows Section 6.7 of Ljungqvist and Sargent (2018).

Note that the cutoff point where the two terms in the parenthesis are equal is unique. This is because p is a linear function of offered price p, so it is increasing and $c + \int_{\underline{p}}^{B} v(p')dH(p')$ is obtained as some constant irrelevant of p. Hence, there is single crossing of these two terms in the parenthesis when the two terms are equal. We define such price as reservation price $\tilde{p}$ from $v(p)$.

The reservation price $\tilde{p}$ is defined as the price at which the consumer is indifferent between buying now and not buying now. That implies that the two values are equal when $p = \tilde{p}$, so $c + \int_{\underline{p}}^{B} v(p')dH(p') = \tilde{p}$. Note that the reservation price property holds here - the consumer choose to buy now if $p \leq \tilde{p}$ because the price offered today is smaller than the expected price and costs that he should pay if he rejects the offer. Using this property, further results can be derived. $E(p)$ below is the unconditional mean of distribution H.

$$\begin{aligned}
\tilde{p} &= c + \int_{\underline{p}}^{B} v(p')dH(p') \\
&= c + \int_{\underline{p}}^{\tilde{p}} p'dH(p') + \int_{\tilde{p}}^{B} \tilde{p}\, dH(p') \\
c &= \int_{\underline{p}}^{\tilde{p}} (\tilde{p} - p)dH(p) = [(\tilde{p} - p)H(p)]_{\underline{p}}^{\tilde{p}} + \int_{\underline{p}}^{\tilde{p}} H(p)dp \\
&= \int_{\underline{p}}^{\tilde{p}} H(p)dp \\
E(p) &= \int_{\underline{p}}^{\tilde{p}} p\, h(p)dp = \int_{\underline{p}}^{\tilde{p}} (1 - H(p))dp - [(1 - H(p)p]_{\underline{p}}^{\tilde{p}} \\
&= \int_{\underline{p}}^{\tilde{p}} (1 - H(p))dp + \underline{p} \\
\therefore \tilde{p} &= c + E(p)
\end{aligned}$$

We used the property that $G(\tilde{p}) = 1$ and $H(\tilde{p}) = 1$ because no seller will offer a price over $\tilde{p}$ as they know that is the consumers' reservation price.

1.2 Sellers

A seller makes offers to ν measure of potential buyers each period, and it knows that νq of them got only one offer and $\nu(1 - q)$ of them got two offers. Total number of offers is $\nu q + 2\nu(1 - q) = \nu(2 - q)$. Hence, it believes that the fraction of its customers who got only one offer is $\hat{q} = \frac{q}{2-q}$. Hence, it believes that $\hat{q}$ of its customers got offer from itself only while $(1 - \hat{q})$ of customers got another offer from a competing firm. It also takes its competitor's price distribution G as given. Now let $Q(p)$ be the probability that the seller's customer will accept an offer $p \leq \tilde{p}$ as below.

$$Q(p) = \hat{q} + (1 - \hat{q})(1 - G(p))$$

The seller seeks to maximize its expected profit per consumer $pQ(p)$.

2 Equilibrium

An equilibrium is defined as the distribution of price offers $G(p)$ on $[\underline{p}, \tilde{p}]$, the distribution of price offers for each consumer $H(p)$ on $[\underline{p}, \tilde{p}]$, and the reservation price $\tilde{p}$ such that

1. $H(p)$ satisfies equation (1).
2. $\tilde{p}$ is an optimal reservation price for buyers satisfying $c = \int_{\underline{p}}^{\tilde{p}} H(p)dp$.
3. Sellers are indifferent with respect to charging any $p \in [\underline{p}, \tilde{p}]$, so they randomize in G.

We characterize an equilibrium by using guess-and-verify method. First, guess $G(p)$ in a following way. Set $\underline{p} = \hat{q}\tilde{p}$ and let $G(p)$ be

$$G(p) = \begin{cases} 0 & \text{if } p < \underline{p} \\ 1 - \frac{\tilde{p}-p}{p}\frac{\hat{q}}{1-\hat{q}} & \text{if } p \in [\underline{p}, \tilde{p}] \\ 1 & \text{if } p > \tilde{p} \end{cases}$$

With this guess, $Q(p) = \frac{\tilde{p}\hat{q}}{p}, \forall p \in [\underline{p}, \tilde{p}]$ and the expected profit per customer is $\tilde{p}\hat{q}$. We see that the expected profit is independent of a seller's choice of p, so the firm is indifferent with any $p \in [\underline{p}, \tilde{p}]$. Hence, our guess of $G(p)$ with $H(p)$ and $\tilde{p}$ that are correspondingly constructed characterize an equilibrium.

Note that with $q \in (0, 1)$, $G(p)$ is strictly concave in p. This implies that sellers are indifferent between being a high-volume, low-price seller and a low-volume, high-price seller following the distribution G.

2.1 Special Cases

2.1.1 Case of $q = 1$

This means that each buyer gets only one offer. Then $G(p)$ defined above collapses to a distribution that the probability mass of p is concentrated on $p = \tilde{p}$. There is no price dispersion and all sellers offer the reservation price to buyers.

2.1.2 Case of $q = 0$

This means that every buyer gets two offers, so $\hat{q} = 0$. This leads to $G(p)$ defined above collapse to a distribution that the probability mass of p is concentrated on $p = \underline{p}$. This implies that there is Bertrand competition between sellers so the price tends to the lower bound of distribution. Since we impose zero marginal cost, $\underline{p} = 0$.

References

Burdett, K., & Judd, K. L. (1983). Equilibrium price dispersion. *Econometrica*, *51*(4), 955 – 969.

Ljungqvist, L., & Sargent, T. J. (2018). *Recursive macroeconomic theory* (Fourth ed.). Cambridge, MA, USA: MIT Press.